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Spacetime and Worldvolume Supersymmetric Super p -Brane Actions *

E. Sezgin[†]

Department of Physics, Texas A&M University, College Station, TX 77843-4242, USA.

ABSTRACT

We review the salient features of spacetime *and* worldvolume supersymmetric super p -brane actions. These are sigma models for maps from a worldvolume superspace to the target superspace. For p -branes, the symmetries of the model depend crucially on the existence of *closed super $(p+1)$ -forms on a worldvolume superspace*, built out of the pull-backs of the Kalb-Ramond super $(p+1)$ -form in target superspace and its curvature. This formulation of super p -branes is usually referred to as the twistor-like formulation.

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1. Introduction

Manifest spacetime supersymmetry is clearly a desirable feature to have in a superstring theory. Although the Green-Schwarz formulation of superstring goes a long way in this direction [1], nobody really knows how to quantize the theory. This problem is not so much due to the fact that we are dealing with string theory. Even the manifestly supersymmetric formulation of a superparticle suffers from this problem. Needless to mention, the manifestly supersymmetric formulation of higher super p -branes [2-4] do not fair any better either.

The difficulty in covariant quantization of super p -branes is essentially due to the fact that the so called κ -symmetry [1] of these theories is an infinitely reducible one. However, this symmetry is needed in order to arrive at non-manifest worldvolume supersymmetry after gauge fixing. It is natural to enquire into the possibility of dispensing with κ -symmetry and building a new kind of action which will have manifest target space *and* worldvolume supersymmetry from the beginning. In such a formulation, one may hope to get rid of the infinite reducibility problem and thus making a progress towards covariant quantization.

There is another outstanding problem in super p -branes which might also be more easily solvable in these new type of formulations, namely, the problem of how to couple Yang-Mills fields to them. In particular, the case of super fivebrane is of great interest, because there is some evidence for its being closely related to the heterotic superstring in ten dimensions [5].

The motivations mentioned above, so far have only led to new and interesting formulations of super p -branes. Unfortunately, we still don't know if the covariant quantization and Yang-Mills coupling problems have solutions in these new formulations. In fact, in the general case, we don't even know what the true physical degrees of freedom are in a physical gauge. Clearly, a lot of work remains to be done. The purpose of this note is to briefly survey what alternative formulations exist, and then focusing on the twistor-like formulation, giving the main results. We shall begin with the massless superparticle (known as the Brink-Schwarz superparticle [6]), and then, after discussing its various twistor formulations [7-12], we move on to massive superparticles which resemble higher super p -branes in many respects. We shall conclude with the summary of results for super p -branes.

2. Brink-Schwarz Superparticle

To illustrate the main ideas involved in manifestly supersymmetric formulation of strings and higher extended objects, it is useful to consider the simplest case of a massless superparticle in ten dimensions. The relevant action is due to Brink and Schwarz [6], and is given by

$$S = \int d\tau \left[P_\mu (\dot{X}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta}) - \frac{1}{2}eP_\mu P^\mu \right], \quad (2.1)$$

where $e(\tau)$ is the einbein on the worldline, (X^μ, θ^α) ($\mu = 0, 1, \dots, 9; \alpha = 1, \dots, 16$) are the coordinates of target superspace, and $\dot{\theta} \equiv d\theta/d\tau$. This action has manifest spacetime *global*

supersymmetry, and a nonmanifest *local* worldline symmetry known as the κ - symmetry given by [1]

$$\begin{aligned}\delta X^\mu &= i\bar{\theta}\Gamma^\mu\delta\theta, \\ \delta\theta &= \Gamma^\mu P_\mu\kappa, \\ \delta e &= 4i\bar{\kappa}\dot{\theta}, \\ \delta P_\mu &= 0.\end{aligned}\tag{2.2}$$

The trouble with this action is that we don't know how to covariantly quantize it. The easiest way to see the problem is to note that the transformation rule for the fermionic variable θ has the zero mode $\kappa_0 = \Gamma^\mu P_\mu\kappa^1$ modulo the zero mode $\kappa_0^1 = \Gamma^\mu P_\mu\kappa^2$ modulo the zero mode $\kappa_0^2 = \Gamma^\mu P_\mu\kappa^3$, ad infinitum. This shows that we are dealing with an infinitely reducible gauge symmetry, requiring the introduction of infinitely many ghost fields. There are ambiguities in calculations involving infinitely many ghost fields, and moreover, additional complications arise due to the fact that the residual gauge symmetries are not completely fixed by the covariant quantization procedure.

Attempts have been made to introduce new variables which would make it possible to reformulate the superparticle action so that, while the physical degrees of freedom are the same, the κ symmetry, and hence the attendant quantization problems are removed. A number of such reformulations invariably involve the so called twistor or twistor-like variables. These variables are closely related to certain representations of appropriate superconformal groups. Ultimately, we will be interested in one such formulation [10] which is the most suitable one for generalization to higher super p -branes. However, in order to stress the differences between various twistor formulations, we shall briefly review three such formulations below [7,9,10].

3. Three Different Twistor Formulations of the Massless Superparticle

(3a) Supertwistor Formulation

This formulation works in dimensions where a superconformal group exists [7]. Hence the restriction to $d \leq 6$. Let us consider, for example, the case of $N = 8$ supersymmetric particle in $d = 3$. The main idea of the supertwistor formulation is to introduce a supertwistor variable, which is simply a finite dimensional representation of the superconformal group, in this case $OSp(8|4)$, and consists of $(\omega_\alpha, \lambda_\alpha, \psi^i)$, where ω_α and λ_α are *commuting* $SL(2, R)$ spinors, while ψ^i is an anticommuting $SO(8)$ vector. These variables allow us to perform the following field redefinitions:

$$\begin{aligned}P^\mu &= \Gamma_{\alpha\beta}^\mu \lambda^\alpha \lambda^\beta, \\ \psi^i &= \theta^{\alpha i} \lambda_\alpha, \\ \omega^\alpha &= X^\mu \Gamma_\mu^{\alpha\beta} \lambda_\beta + i\theta^{\alpha i} \psi_i.\end{aligned}\tag{3.1}$$

It should be noted that the variable P^μ as defined above, satisfies the constraint $P^\mu P_\mu = 0$. In terms of the supertwistor variables, one can write down the action

$$S = \int d\tau (\dot{\omega}^\alpha \lambda_\alpha + i\dot{\psi}^i \psi_i).\tag{3.2}$$

We no longer need to consider the κ symmetry of this action, because, as one can easily check, the supertwistor variables themselves are κ invariant. Furthermore one can check that this action produces the first two terms of the Brink-Schwarz action (2.1), while the consequence of the last term, namely $P^\mu P_\mu = 0$ is automatically satisfied due to the supertwistor form of the variable P^μ .

The above action has a very simple form, and it can actually be covariantly quantized [7]. However, the cases of $d = 4$ and $d = 6$ are more complicated, because new type of local bosonic symmetries emerge [7,8]. Moreover, the above formalism doesn't generalize to $d = 10$, since there is no superconformal group in ten dimensions.

(3b) Supertwistor-like Formulation

Although a superconformal group doesn't exist in ten dimensions, one may nonetheless try to use variables similar to the supertwistor variables described in the previous section. This has been done by Berkovits [9] who used the following variables

$$\begin{aligned} P^\mu &= \lambda^\alpha \Gamma_{\alpha\beta}^\mu \lambda^\beta , \\ \psi^\mu &= \lambda^\alpha \Gamma_{\alpha\beta}^\mu \theta^\beta , \\ \omega^\alpha &= X^\mu \Gamma_\mu^{\alpha\beta} \lambda_\beta - i\psi^\mu \Gamma_\mu^{\alpha\beta} \theta_\beta , \end{aligned} \tag{3.3}$$

Note the similarity with the set of variables defined in (3.1). These variables, however, don't correspond to a superconformal group, and therefore, we shall refer to them as supertwistor-like variables. Unlike the supertwistors, they are only partially κ -invariant. In terms of these variables, Berkovits' action reads

$$\begin{aligned} S = \int d\tau [& -2\omega^\alpha \dot{\lambda}_\alpha - 2i\psi^\mu \dot{\psi}_\mu + h\psi^\mu \bar{\lambda} \Gamma_\mu \lambda \\ & + h_\alpha (\bar{\lambda} \Gamma^\mu \lambda \Gamma_\mu^{\alpha\beta} \omega_\beta - 2\bar{\lambda} \omega \lambda^\alpha + 2i\psi^\mu \psi^\nu \Gamma_{\mu\nu}^{\alpha\beta} \lambda_\beta)] , \end{aligned} \tag{3.4}$$

where h and h_α are Lagrange multiplier fields. While the covariant quantization of this model is possible [9], its generalization to string theory is not so obvious. It turns out that one way to achieve this is to do away with ω_α and ψ^μ type variables, but keeping the variable λ_α which continues to play a central role in passing to a formulation where κ symmetry is traded for worldline supersymmetry. This is achieved in the twistor-like formulation described below [9].

(3c) Twistor-like Formulation

This formulation is due to Sorokin, Tkach, Volkov and Zheltuhkin [10], and it maintains only the twistor-like variable λ , as defined in (3.3). The advantage of doing so will become clear below. For simplicity, let us focus on the case of $d = 3, N = 1$ massless superparticle for now. The Brink-Schwarz action is replaced by

$$\int d\tau P_\mu \left(\dot{X}^\mu - i\bar{\theta} \Gamma^\mu \dot{\theta} + \bar{\lambda} \Gamma^\mu \lambda \right) . \tag{3.5}$$

From the λ equation of motion, $P^\mu \Gamma^\mu \lambda = 0$, one finds the solution $P_\mu = \bar{\lambda} \Gamma_\mu \lambda$, which satisfies $P^\mu P_\mu = 0$, thanks to the identity: $\Gamma_{\mu(\alpha\beta} \Gamma_{\gamma\delta)}^\mu = 0$. This action has *worldline local $n=1$ supersymmetry* which replaces the κ symmetry. The $n = 1$ supersymmetry closes on-shell. To close it off-shell, one introduces superfields on the world superline as follows [10]:

$$\begin{aligned} P_\mu(\tau, \eta) &= P_\mu(\tau) + i\eta Q_\mu(\tau) , \\ X^\mu(\tau, \eta) &= X^\mu(\tau) + iY^\mu(\tau) , \end{aligned} \quad (3.6)$$

where (Q_μ, Y^μ) are auxiliary fields and η is the fermionic coordinate in the $n = 1$ worldline superspace. We can view the twistor variable λ as the superpartner of the target superspace fermionic coordinate θ and define

$$\theta(\tau, \eta) = \theta(\tau) + \eta \lambda(\tau) . \quad (3.7)$$

Then, the off-shell version of the action (3.5) can be written as a superspace integral [10]

$$S \int = -id\tau d\eta P_\mu (DX^\mu + i\bar{\theta} \Gamma^\mu D\theta) , \quad (3.8)$$

where $D = \frac{\partial}{\partial \eta} + i\eta \frac{\partial}{\partial \tau}$. One can furthermore combine $X^\mu(\tau, \eta)$ and $\theta(\tau, \eta)$ to define target superspace coordinates, which are worldline superfields.

Note that the action (3.5) is linear in time derivative and doesn't contain the einbein. Thus, it has the form of a Wess-Zumino term. Moreover, it turns out that this form of the action does admit generalization to higher $n(N)$ supersymmetry, curved superspace as well as superstrings and higher super p -branes. First, let us describe the higher $n(N)$ supersymmetry.

A convenient notation for dealing with two superspaces is as follows. We use the same letters for worldline and target superspaces, but distinguish the two by underlying the target superspace coordinates. The notation for general super p -branes can be summarized as follows:

Worldline superspace:	$\mathcal{M} :$	$Z^M = (X^m, \theta^\mu)$
Target superspace:	$\underline{\mathcal{M}} :$	$Z^{\underline{M}} = (X^{\underline{m}}, \theta^{\underline{\mu}})$
Worldline supervielbein:	$E_M^A :$	$A = (a, \alpha)$
Worldline supervielbein:	$E_{\underline{M}}^{\underline{A}} :$	$\underline{A} = (\underline{a}, \underline{\alpha})$

For the superparticle the target superspace notation is as above, but for the worldline superspace we use the notation, $Z^M = (\tau, \theta^\mu)$, $A = (0, r)$, $(\mu, r = 1, \dots, n)$. For super p -branes, on the other hand, the range of indices are as follows: $m, a = 0, 1, \dots, p$; $\mu, \alpha = 1, \dots, n$; $\underline{m}, \underline{a} = 0, 1, \dots, d-1$; $\underline{\mu}, \underline{\alpha} = 1, \dots, MN$, where M is the dimension of the minimum dimensional spinor representation of $SO(d-1, 1)$ and N is the dimension of the defining representation of the automorphism group \underline{G} of the super Poincaré algebra in d dimensions. It is important to note that the number of worldvolume supersymmetries is half of the target space supersymmetries (counted in terms of worldvolume spinors), i.e. $n = \frac{1}{2}MN$. A further notation is that when the automorphism group is nontrivial, the index α represents

a pair of indices $\alpha'r$. Thus, $\Gamma_{\alpha\beta}^a = \Gamma_{\alpha'\beta'}^a \eta_{rs}$, where η_{rs} is the invariant tensor of the automorphism group G of the worldvolume super Poincaré algebra. For further aspects of this notation, see the Table.

Let us now consider the case of $n = 8$ supersymmetric massless superparticle [12,13]. The twistor-like variable λ_r^α satisfies the constraint

$$\underline{\lambda}_r \Gamma^a \lambda_s = \frac{1}{8} \delta_{rs} (\bar{\lambda}_q \Gamma^a \lambda_q) . \quad (3.9)$$

We use a notation in which the contracted $\underline{\alpha}$ indices are suppressed, and the parenthesis such as those in (3.9) indicate these contractions. It is useful to express the constraint (3.9) in geometrical way. To this end, let us define

$$E_A^{\underline{A}} = E_A^M (\partial_M Z^{\underline{M}}) E_{\underline{M}}^{\underline{A}} . \quad (3.10)$$

We can make the identifications

$$E_r^\alpha|_{\theta=0} = \lambda_r^\alpha , \quad E_0^a|_{\theta=0} = \mathcal{E}_0^a , \quad (3.11)$$

where $\mathcal{E}_0^a = \dot{X}^a - i\bar{\theta}\Gamma^a\dot{\theta}$. Thus, we have the expansion $\theta^\alpha(\tau, \theta) = \theta^\alpha(\tau) + \lambda_r^\alpha(\tau)\theta^r + \dots$. The action can now be written as follows

$$S = \int d\tau d^8\theta P_{\underline{a}}^r E_r^{\underline{a}} . \quad (3.12)$$

The field equation for the Lagrange multiplier superfield $P_{\underline{a}}^r$ is

$$E_r^{\underline{a}} = 0 . \quad (3.13)$$

Taking the supercurl of this equation, we arrive at the integrability condition

$$(E_r \Gamma^a E_s) = \delta_{rs} E_0^a . \quad (3.14)$$

the lowest component of which implies the constraint (3.9). In deriving (3.14), the following Lemma is useful:

$$D_A E_B^{\underline{C}} - (-1)^{AB} D_B E_A^{\underline{C}} = -T_{AB}^C E_C^{\underline{C}} + (-1)^{A(B+\underline{D})} E_B^{\underline{D}} E_A^{\underline{E}} T_{\underline{ED}}^{\underline{C}} , \quad (3.15)$$

where the covariant derivative $D_A = E_A^M D_M$ rotates the indices A and \underline{A} and the tangent space components of the supertorsion $T_{MN}^C = \partial_M E_N^C + \Omega_M^{CD} E_{ND} - (-1)^{MN}(M \leftrightarrow N)$ are defined as: $T_{AB}^C = (-1)^{A(B+N)} E_B^N E_A^M T_{MN}^C$, and similarly for $T_{\underline{AB}}^{\underline{C}}$.

There may seem to be many more fields in the θ -expansion of the action functional in (3.12). However, as shown in [12,13], there are many redundant fields, and the action (3.12) is classically equivalent to the Brink-Schwarz superparticle action. For a detailed discussion of this equivalence as well as the local supersymmetry of the action and its relation to κ -symmetry, we refer the reader to [12,13].

One immediate bonus that follows from the above formulation is that its generalization to curved superspace is immediate. We simply elevate the supervielbeins occuring in the action formula to those of curved superspace. The action then has precisely the same form as in the flat superspace. As for the twistor constraint (3.14), it will now follow from the integrability condition of (3.13), followed by imposition of a suitable set of supertorsion constraints (both in worldline and target superspaces). In what follows, our strategy will be to fix the geometry of the worldvolume and target superspaces from the beginning, though one may try to determine them, at least partially, from other considerations such as worldvolume supersymmetry.

Another bonus of the twistor-like formulation is that it generalizes quite naturally to superstrings [13,14] and higher super p -branes [15,16]. The case of heterotic string has been discussed in great detail in Refs. [13,14]. Here, we shall focus on the results for higher super p -branes [15,16]. Since higher super p -branes resemble in many respects the massive superparticle, we shall first describe the latter case, and then give the result for the general super p -branes. For an alternative and distinct approach to twistor-like formulation of super p -branes, see Ref. [17].

4. Twistor-like Formulation of the Massive Superparticle

The usual κ invariant massive superparticle action is given by

$$S = \int d\tau \left(\frac{1}{2} e^{-1} \mathcal{E}_\tau^a \mathcal{E}_\tau^a + \frac{1}{2} e + \mathcal{E}_\tau^{\underline{A}} B_{\underline{A}} \right), \quad (4.1)$$

where e is the einbein on the worldline, $\mathcal{E}_\tau^{\underline{A}} = \partial_\tau Z^{\underline{M}} E_{\underline{M}}^{\underline{A}}$ and $B_{\underline{A}} = E_{\underline{A}}^{\underline{M}} B_{\underline{M}}$. The latter is the super one-form that is analogous to the Kalb-Ramond field in string theory. It is needed for the κ -symmetry of the action. This symmetry imposes some constraints on the supertorsion as well as $H = dB$. The form of these constraints and the κ symmetry transformation rules can be found in [18]. Here, we shall make further choices regarding the form of the constraints, in order to fix the target superspace geometry as much as possible. We will work with the following constraints

$$\begin{aligned} T_{\underline{\alpha}\underline{\beta}}^{\underline{c}} &= -2i(\Gamma^{\underline{c}})_{\underline{\alpha}\underline{\beta}}, \\ T_{\underline{\alpha}\underline{b}}^{\underline{c}} &= 0, \\ H_{\underline{\alpha}\underline{\beta}} &= -2iC_{\underline{\alpha}\underline{\beta}}, \\ H_{\underline{\alpha}\underline{a}} &= 0. \end{aligned} \quad (4.2)$$

For later purpose, it is also useful to give the Nambu-Goto form of the action, which is obtained from (4.1) by eliminating the einbein through its equation of motion

$$S = \int d\tau \left[(\mathcal{E}_\tau^a \mathcal{E}_\tau^a)^{1/2} + \mathcal{E}_\tau^{\underline{A}} B_{\underline{A}} \right]. \quad (4.3)$$

In order to pass to the twistor-like formulation, we adopt the target superspace constraints (4.2), and elevate the worldline to an $N = 8$ superspace with the following constraints

$$\begin{aligned} T_{rs}{}^0 &= -2i\delta_{rs} , \\ T_{0r}{}^0 &= 0 , \\ T_{s0}{}^r &= 0 , \\ T_{rs}{}^q &= 0 . \end{aligned} \tag{4.4}$$

The rationale behind these constraints is that they still leave room for $N = 8$ local superdiffeomorphisms [12,13]. Having now specified the geometry of worldline and target superspaces, in addition to the superfields occuring in the action (3.12) and the super one-form B_M , we introduce two Lagrange multiplier superfields P^M and Q , and propose the following action [16]

$$S = \int d\tau d^8\theta \left[P_{\underline{a}}^r E_r^{\underline{a}} + P^M (\tilde{B}_M - \partial_M Q) \right] , \tag{4.5}$$

where the super one-form \tilde{B} is defined as

$$\tilde{B}_M = \partial_M Z^{\underline{M}} B_{\underline{M}} - \frac{i}{16} E_M^0 H_{rr} , \tag{4.6}$$

and $H_{rr} = E_r^{\underline{A}} E_r^{\underline{B}} H_{\underline{B}\underline{A}}$, $H_{\underline{B}\underline{A}}$ are the tangent space components of the field strength $H = dB$: $H_{\underline{A}\underline{B}} = (-)^{\underline{A}(\underline{B}+\underline{N})} E_{\underline{B}}^{\underline{N}} E_{\underline{A}}^{\underline{M}} H_{\underline{M}\underline{N}}$, where the indices in the exponent indicate Grassmannian parities. Recall that $\underline{M} = (\tau, \mu)$, $\underline{A} = (0, r)$, $\underline{M} = (\underline{m}, \underline{\mu})$ and $\underline{A} = (\underline{a}, \underline{\alpha})$. The indices of the bosonic (fermionic) coordinates have the parity 0(1).

The form of the action (4.5) is inspired by results of Refs. [12,13] for massless superparticle and heterotic superstring. Note that the independent world-line *superfields* in the action are: $P_{\underline{a}}^r$, P^M , Q , E_M^0 and $Z^{\underline{M}}$. An important property of the action (4.5) is that it is invariant under $n = 8$ local world-line supersymmetry, as opposed to the κ -symmetry. (The latter emerges as a special case of the former in a certain gauge). The supersymmetry of the second and third terms in the action is manifest (everything transform like supertensors), while the supersymmetry of the first term is due to the fact that $E_r^{\underline{a}}$ transforms homogeneously like D_r does, and this can be compensated by a suitable transformation of the Lagrange multiplier.

The field equation for $P_{\underline{a}}^r$ yields, as before, Eq. (3.13), and as an integrability condition, the twistor constraint (3.14). The field equation for P^M implies that $\tilde{B}_M = \partial_M Q = 0$, from which it follows that $d\tilde{B} \equiv \tilde{H} = 0$. Using the constraints (4.2), (4.4) and (3.13), one can show that this constraint is indeed satisfied. In fact, the form of \tilde{B} is engineered precisely such that $d\tilde{B} = 0$, modulo the constraints (4.2), (4.4) and (3.13).

As a consequence of $d\tilde{B} = 0$, the action (4.5) has also the gauge invariance

$$\delta P^M = \partial_N \Lambda^{NM} , \tag{4.7}$$

where Λ^{MN} is an arbitrary graded antisymmetric superfield. In showing this one uses the fact that $d\tilde{B} = 0$, which in turn requires the use of the constraint (3.13), which is

the equation of motion for $P_{\underline{a}}^r$. Of course, one is not supposed to use field equations in showing gauge invariance. However, here we are allowed to do so, because in showing the invariance, the terms that are proportional to the field equation of $P_{\underline{a}}^r$ can always be cancelled by an appropriate variation of $P_{\underline{a}}^r$.

The action (4.5) has the additional gauge invariance

$$\delta P_{\underline{a}}^r = D_q(\xi^{qrs}\Gamma_{\underline{a}}E_s) , \quad \delta P^M = -E_r^M D_q(\xi^{qrs}E_s) , \quad (4.8)$$

where the parameter $\xi_{\underline{\alpha}}^{qrs}(\tau, \theta)$ is totally symmetric and traceless in its worldline indices, and we have used the constraints (4.2), (4.4), (3.13) and assumed the existence of the Dirac matrix identity

$$\Gamma_{\underline{\alpha}\underline{\beta}}^{\underline{a}}\Gamma_{\underline{\gamma}\underline{\delta}}^{\underline{a}} + C_{\underline{\alpha}\underline{\beta}}C_{\underline{\gamma}\underline{\delta}} + \text{cyclic}(\underline{\alpha}\underline{\beta}\underline{\gamma}) = 0 . \quad (4.9)$$

Among the spaces listed in the Table, this identity holds in $d = 5, 9$. The gauge invariance (4.8) plays an important role in getting rid of many redundant fields and thus in showing the classical equivalence of the action (4.5) with the usual action (4.1).

Lets us now consider the remaining equations of motion. The field equation of Q reads $\partial_M P^M = 0$. This equation has the solution [13] $P^M = \partial_N \Sigma^{NM} + \theta^8 \delta_\tau^M T$, where T is a constant and Σ^{MN} is an arbitrary graded antisymmetric superfield, which can be gauged away by using the gauge symmetry (4.7). Substituting this algebraic solution into the action (4.5) and after some algebra, one can show that the action reduces to

$$S = \int d\tau \left[p_{\underline{a}}(\mathcal{E}_0^{\underline{a}} - \frac{1}{8}\bar{\lambda}_r \Gamma^{\underline{a}} \lambda_r) + \partial_\tau Z^{\underline{M}} B_{\underline{M}} + (\mathcal{E}_\tau^{\underline{a}} \mathcal{E}_\tau^{\underline{a}})^{1/2} \right] , \quad (4.10)$$

where $p_{\underline{a}} = (D^\tau)_r P_{\underline{a}}^r|_{\theta=0}$. With arguments parallel to those of [12,13], we expect that the Lagrange multiplier $p_{\underline{a}}$ does not describe any new degree of freedom, and the field equations of (4.1) and (4.5) are classically equivalent.

The key ingredient in the above formulation of massive superparticle was the existence a super one-form \tilde{B} on the worldline such that $d\tilde{B} = 0$ modulo an acceptable set of worldline and target superspace constraints. Therefore, in order to generalize the above construction to higher super p -branes, it is natural to search for closed $(p+1)$ -forms in the worldvolume superspace together with a suitable set of constraints in worldvolume and target superspaces. Indeed, in [16] we found such superforms and we were able to give a general construction of the twistor-like super p -brane actions, generalizing a result of [15] for the case of supermembrane. In the remainder of this review, we shall summarize the result for general super p -branes.

5. Twistor-like Formulation of Super p -Branes

In accordance with the procedure described earlier, we first fix the supergeometry of the worldvolume superspace. In analogy with (3.1), we impose the following constraints

$$T_{\alpha\beta}{}^a = -2i(\Gamma^a)_{\alpha\beta}, \quad T_{b\alpha}{}^a = 0, \quad T_{bc}{}^a = 0, \quad T_{\alpha\beta}{}^\gamma = 0 . \quad (5.1)$$

See the Table for the symmetry properties of the gamma matrices. We also fix the target superspace geometry. As for the target superspace geometry, in addition to the superstoration, we need to consider the Kalb-Ramond type super $p + 1$ form B with field strength $H = dB$. In analogy with (4.2), we then choose the following constraints [3]

$$\begin{aligned} T_{\underline{\alpha}\underline{\beta}}^{\underline{c}} &= -2i(\Gamma^{\underline{c}})_{\underline{\alpha}\underline{\beta}} , & T_{\underline{b}\underline{\alpha}}^{\underline{a}} &= 0 , & T_{\underline{\alpha}\underline{\beta}}^{\underline{\gamma}} &= 0 , \\ H_{\underline{\alpha}\underline{\beta}\underline{c}_1 \dots \underline{c}_p} &= i\xi^{-1}(\eta \Gamma_{\underline{c}_1 \dots \underline{c}_p})_{\underline{\alpha}\underline{\beta}} , & H_{\underline{\alpha}\underline{b}_1 \dots \underline{b}_{p+1}} &= 0 , & H_{\underline{\alpha}\underline{\beta}\underline{\gamma} \dots \underline{A}_1 \dots \underline{A}_{p-1}} &= 0 , \end{aligned} \quad (5.2)$$

where $\xi = (-)^{(p-2)(p-5)/4}$ and η is a matrix chosen such that $\eta \Gamma_{\underline{c}_1 \dots \underline{c}_p}$ is symmetric. $\eta = 1$ except for the following cases: $\eta = \Gamma_{d+1}$ for $(p = 3, d = 8)$, with the definition $\Gamma_{d+1} = \Gamma_0 \Gamma_1 \dots \Gamma_{d-1}$, and $\eta = 1 \times \sigma_2$ for $(p = 2, d = 5)$. See the Table for further information on the notation and properties of the Dirac matrices in diverse dimensions.

In $d = 11$ dimensions the above constraints describe the $d = 11$ supergravity theories. In other cases, a detailed analysis of the constraint remains to be carried out. Presumably, they describe supergravity theories containing $(p + 1)$ -form potentials.

Having specified the geometry of the worldvolume and target superspaces, our next goal is to write down an action for twistor-like super p -branes in analogy with the action (4.5). Such an action was proposed in [15] for the case of the supermembrane. In [16] we generalized that result and proposed the following action for all super p -branes

$$S = \int d^{p+1} \sigma d^{mn} \theta \left[P_{\underline{a}}^{\alpha} E_{\alpha}^{\underline{a}} + P^{M_1 \dots M_{p+1}} (\tilde{B}_{M_1 \dots M_{p+1}} - \partial_{M_1} Q_{M_2 \dots M_{p+1}}) \right] , \quad (5.3)$$

where $P_{\underline{a}}^{\alpha r}$, $P^{M_1 \dots M_{p+1}}$ and $Q_{M_1 \dots M_p}$ are Lagrange multiplier superfields (the latter two are graded totally antisymmetric) and the $(p + 1)$ -form \tilde{B} is given by [16]

$$\begin{aligned} \tilde{B}_{M_1 \dots M_{p+1}} &= (-1)^{\epsilon_{p+1}(M, \underline{M})} \partial_{M_{p+1}} Z^{\underline{M}_{p+1}} \dots \partial_{M_1} Z^{\underline{M}_1} B_{\underline{M}_1 \dots \underline{M}_{p+1}} \\ &\quad - \frac{i}{2mn(p+1)} \Gamma_{c_{p+1}}^{\alpha\beta} \left(E_{M_{p+1}}^{c_{p+1}} \dots E_{M_1}^{c_1} H_{\alpha\beta c_1 \dots c_p} + \text{cyclic } [M_1 \dots M_{p+1}] \right) . \end{aligned} \quad (5.4)$$

The grading factor is given by $\epsilon_{p+1}(M, \underline{M}) = \sum_{n=1}^p (M_1 + \dots M_n)(M_{n+1} + \underline{M}_{n+1})$, and the pullback of H by

$$H_{A_1 \dots A_{p+2}} = (-1)^{\epsilon_{p+2}(A, \underline{A})} E_{A_{p+2}}^{\underline{A}_{p+2}} \dots E_{A_1}^{\underline{A}_1} H_{\underline{A}_1 \dots \underline{A}_{p+2}} . \quad (5.5)$$

The field equation for $P_{\underline{a}}^{\alpha}$ is

$$E_{\alpha}^{\underline{a}} = 0 . \quad (5.6)$$

The integrability condition for this equation yields the analog of the twistor constraint (3.14) for super p -branes, and it takes the form

$$(E_{\alpha} \Gamma^{\underline{a}} E_{\beta}) = \Gamma_{\alpha\beta}^{\underline{a}} E_{\underline{a}} . \quad (5.7)$$

The field equation for $P^{M_1 \dots M_{p+1}}$ is $\tilde{H}_{M_1 \dots M_{p+2}} = \partial_{M_1} \tilde{B}_{M_2 \dots M_{p+2}} + \text{cyclic } [M_1 \dots M_{p+2}] = 0$. Given \tilde{B} as in (5.4), it is nontrivial to show that this equation holds. A tedious calculation, which can be found in [16] and we will not reproduce here, shows that this closure property indeed holds for the cases $(p, m, n) = (2, 2, 8), (5, 4, 2), (2, 2, 4), (3, 4, 1), (2, 2, 2)$ and $(2, 2, 1)$ (See the Table). The $p = 2$ cases were already considered in [15]. In these calculations, the following Dirac matrix identity plays an important role [3,4]

$$\Gamma_{(\underline{\alpha}\underline{\beta}}^c (\eta \Gamma^{\underline{cc}_1 \dots \underline{c}_{p-1}})_{\underline{\gamma}\underline{\delta}}) = 0. \quad (5.8)$$

Since the equation $d\tilde{B} = 0$ holds, the analog of the gauge invariance (4.7) exists also for super p -branes, and reads: $\delta P^{M_1 \dots M_{p+1}} = \partial_N \Sigma^{NM_1 \dots M_{p+1}}$, where the parameter is completely graded antisymmetric. Using this symmetry, the field equation for $Q^{M_1 \dots M_p}$: $\partial_{M_1} P^{M_1 \dots M_{p+1}} = 0$, can be put into the form $P^{M_1 \dots M_{p+1}} = T \epsilon^{m_1 \dots m_{p+1}} \delta_{m_1}^{M_1} \dots \delta_{m_{p+1}}^{M_{p+1}} \theta^{mn}$, where T is constant. Substituting this into the action and after considerable amount of algebra which has been described in [16], one finds the result

$$S = \int d^{p+1} \sigma d^{mn} \theta P_{\underline{a}}^{\alpha} E_{\alpha}^{\underline{a}} + \frac{(p+1)!}{2} \int d^{p+1} \sigma (-\det E_m^{\underline{a}} E_n^{\underline{a}})^{1/2} |_{\theta=0} \\ + \int d^{p+1} \sigma \epsilon^{m_1 \dots m_{p+1}} \partial_{m_{p+1}} Z^{\underline{M}_{p+1}} \dots \partial_{m_1} Z^{\underline{M}_1} B_{\underline{M}_1 \dots \underline{M}_{p+1}} |_{\theta=0}, \quad (5.9)$$

Going back to the original form of the action, the field equation for $Z^{\underline{M}}$ derived from it, may seem to describe a large number of degrees of freedom. However, one expects a number of gauge invariances, similar to (4.8), which ought to play an important role in reducing drastically the true number of degrees of freedom. In fact in [15], such gauge invariances have been proposed for the case of supermembranes ($p = 2$), and it has been claimed that the true degrees of freedom are those that follow from the usual κ -symmetric formulation of the supermembrane. We have not checked this, and we don't know yet what the full set of gauge symmetries involving the Lagrange multiplier fields are, and consequently we don't know yet what the true degrees of freedom are for general super p -branes.

6. Conclusions and Open Problems

The main result concerning the twistor-like formulation of super p -branes is the action (4.5), together with the definition (5.3), or alternatively the formula (5.9). The latter form of the action coincides with the Nambu-Goto form of the usual super p -brane action. The difference is due to the Lagrange multiplier term. It is not altogether clear whether the equations of motions are equivalent to those which follow from the usual super p -brane action. For this to happen, one must show that there is sufficiently powerful gauge symmetry of the action which makes it possible to gauge away the Lagrange multiplier. We have shown that for the massive superparticle such a gauge symmetry indeed exists. The existence of this gauge symmetry relies on the Dirac matrix identity (4.9). It remains to be seen whether a similar gauge symmetry exists for other values of p . We expect that

the p -brane Dirac matrix identity (5.8) will play an essential role in proving the existence of such a symmetry.

One of the essential ingredients of the twistor-like transform is the existence of a closed super $(p+1)$ -form on the worldvolume superspace which is constructed out of the pull-backs of a super $(p+1)$ -form and its curvature in target superspace. We have shown that this closed $(p+1)$ -form exists for the cases $(p, m, n) = (2, 2, 8), (5, 4, 2), (2, 2, 4), (3, 4, 1), (2, 2, 2)$ and $(2, 2, 1)$. The $p = 2$ cases were considered in [15]. We believe that the existence of this closed $(p+1)$ -form should have some interesting geometric interpretation, independent of the role it plays in the twistor-like transform. For instance, it seems that it is related to the light-like integrability principle [19,13].

There are a number of open problems which deserve further investigation. Some of these problems are:

- (1) What is the full set of symmetries of the action and what are the physical degrees of freedom?
- (2) What is the precise relation between our action and the usual one [3] at the quantum level?
- (3) Can the quantization problems of the usual κ -symmetric action be avoided by the new action?
- (4) Are the symmetries of the action anomaly-free?
- (5) Is the twistor-like formulated of super p -brane theory finite? Can one have a handle on this problem, at least at the perturbative level?
- (6) How can we couple Yang-Mills sector to super p -branes? (Such theories are usually referred to as heterotic p -brane theories, because of their similarity to the heterotic string theory).

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TABLE

Target Space Data								
d	11	10	9	8	7	6	5	4
(M, N)	(32,1)	(16,1)	(16,1)	(16,1)	(8,2)	(4,2)	(4,2)	(4,1)
\underline{G}	–	–	–	–	USp(2)	USp(4)	USp(4)	SO(4)
$C_{\alpha'\beta'}$	A	A	S	S	S	S	S	A
$\Gamma_{\alpha'\beta'}^a$	S	S	S	S	A	A	A	S
η_{rs}	–	–	–	–	A	A	A	S
Type	M	MW	PM	PM	SM	SMW	SM	M
Worldvolume Data ($p \geq 2$)								
p	2	5	4	3	2	3	2	2
(m, n)	(2,8)	(4,2)	(4,2)	(4,2)	(2,4)	(4,1)	(2,2)	(2,1)
G	SO(8)	USp(2)	USp(2)	SO(2)	SO(4)	–	SO(2)	–
$C_{\alpha'\beta'}$	A	S	S	A	A	A	A	A
$\Gamma_{\alpha'\beta'}^a$	S	A	A	S	S	S	S	S
η_{rs}	S	A	A	S	S	–	S	–
Type	M	SMW	SM	M	M	M	M	M

In this table, d indicates the dimension of spacetime, M is the dimension of the spinor irrep of $SO(d-1, 1)$, N is the dimension of the defining representation of the automorphism group \underline{G} of the super Poincaré algebra in d dimensions, $C_{\alpha'\beta'}$ is the charge conjugation matrix, $\Gamma_{\alpha'\beta'}^a$ are the Dirac matrices $(\Gamma^a C)_{\alpha'\beta'}$ and η_{rs} is the invariant tensor of \underline{G} . We often use the notation in which a pair of indices $(\alpha' r)$ is replaced by a single index $\underline{\alpha}$. Furthermore, in $d = 6, 10$ the matrices $\Gamma_{\underline{\alpha}\underline{\beta}}^a$ are chirally projected Dirac matrices and $\Gamma_{\underline{a}}^{\alpha\beta}$ are projected with opposite chirality. In this notation raising or lowering of the spinor indices is not needed. The types of spinors are characterized according to the reality and chirality conditions imposed on them, namely Majorana (M), pseudo-Majorana (PM), symplectic Majorana (SM), Majorana-Weyl (MW) and symplectic Majorana-Weyl (SMW). Corresponding quantities are listed for the super p -branes that arise in target space dimension d .